TRU Math: Teaching for Robust Understanding in Mathematics[1]

Scoring Rubric

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This document provides the summary scoring rubric for the TRU Math (Teaching for Robust Understanding of Mathematics) classroom analysis scheme. TRU Math addresses five general dimensions of mathematics classroom activity, and one dimension that is algebra-specific. Each of these six dimensions is coded separately during whole class discussions, small group work, student presentations, and individual student work.

1. The Mathematics	2. Cognitive Demand	3. Access to Mathematical Content	4. Agency, Authority, and Identity	5. Uses of Assessment
The extent to which the mathematics discussed in the observed lesson is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained	*The extent to which classroom interactions create and maintain an environment of productive intellectual challenge that is conducive to students' mathematical development*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class*	*The extent to which students have opportunities to conjecture, explain, make mathematical arguments, and build on one another's ideas, in ways that contribute to students' development of agency, authority, and their identities as doers of mathematics*	*The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings*

Content Elaboration for Contextual Algebraic Tasks: - The extent to which students are supported in dealing with complex modeling and applications problems, which typically call for understanding complex problem contexts (most frequently described in text), identifying relevant variables and the relationships between them, representing those variables and relationships symbolically, operating on the symbols, and interpreting the results.

This document is a research tool; it is not intended for use in teacher evaluations. Detailed instructions regarding the use of this scoring rubric are provided in *The TRU Math Scoring Guide.* Information regarding the genesis, rationale, and applications of the TRU Math scheme can be found in the document *An Introduction to Teaching for Robust Understanding in Mathematics (TRU Math)*. Both documents, along with this scoring rubric and TRU Math coding sheets, are available at <http://ats.berkeley.edu/tools.html>.

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Summary Rubric

	The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
	How accurate, coherent, and well justified is the mathematical content?	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent does the teacher support access to the content of the lesson for all students?*	*To what extent are students the source of ideas and discussion of them? How are student contributions framed?*	*To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas when potentially valuable or address misunderstandings when they arise?*
1	Classroom activities are unfocused or skills-oriented, lacking opportunities for engagement with key grade level content (as specified in the Common Core Standards)	Classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises.	There is differential access to or participation in the mathematical content, and no apparent efforts to address this issue.	The teacher initiates conversations. Students' speech turns are short (one sentence or less), and constrained by what the teacher says or does.	Student reasoning is not actively surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
2	Activities are at grade level but are primarily skills-oriented, with few opportunities for making connections (e.g., between procedures and concepts) or for mathematical coherence (see glossary).	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges, removing opportunities for productive struggle.	There is uneven access or participation but the teacher makes some efforts to provide mathematical access to a wide range of students.	Students have a chance to explain some of their thinking, but the teacher is the primary driver of conversations and arbiter of correctness. In class discussions, student ideas are not explored or built upon.	The teacher refers to student thinking, perhaps even to common mistakes, but specific students' ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
3	Classroom activities support meaningful connections between procedures, concepts and contexts (where appropriate) and provide opportunities for building a coherent view of mathematics.	The teacher's hints or scaffolds support students in productive struggle in building understandings and engaging in mathematical practices.	The teacher actively supports and to some degree achieves broad and meaningful mathematical participation; **OR** what appear to be established participation structures result in such engagement.	Students explain their ideas and reasoning. The teacher may ascribe ownership for students' ideas in exposition, **AND/OR** students respond to and build on each other's ideas.	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

Whole Class Activities: **L**aunch, Teacher **E**xposition, Whole Class **D**iscussion

*On the score sheet, Circle one of **L / E / D** if the episode is primarily of that type. If a **L**aunch is primarily logistical, some dimensions may be labeled N/A.*

	The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
	How accurate, coherent, and well justified is the mathematical content?	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent does the teacher support access to the content of the lesson for all students?*	*To what extent are students the source of ideas and discussion of them? How are student contributions framed?*	*To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas when potentially valuable or address misunderstandings when they arise?*
1	Classroom activities are unfocused or skills-oriented, lacking opportunities for engagement with key grade level content (as specified in the Common Core Standards)	Classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises.	There is differential access to or participation in the mathematical content, and no apparent effort to address this issue.	The teacher initiates conversations. Students' speech turns are short (one sentence or less), and constrained by what the teacher says or does.	Student reasoning is not actively surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
2	Activities are at grade level but are primarily skills-oriented, with few opportunities for making connections (e.g., between procedures and concepts) or for mathematical coherence (see glossary).	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges, removing opportunities for productive struggle.	There is uneven access or participation, but the teacher makes some efforts to provide mathematical access to a wide range of students.	Students have a chance to explain some of their thinking, but the teacher is the primary driver of conversations and arbiter of correctness. In class discussions, student ideas are not explored or built upon.	The teacher refers to student thinking, perhaps even to common mistakes, but specific students' ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
3	Classroom activities support meaningful connections between procedures, concepts and contexts (where appropriate) and provide opportunities for building a coherent view of mathematics.	The teacher's hints or scaffolds support students in productive struggle in building understandings and engaging in mathematical practices.	The teacher actively supports and to some degree achieves broad and meaningful mathematical participation; **OR** what appear to be established participation structures result in such engagement.	Students explain their ideas and reasoning. The teacher may ascribe ownership for students' ideas in exposition, **AND/OR** students respond to and build on each other's ideas.	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

Small Group Work

If students are engaged in early brainstorming, the role of the teacher is to support students in exploring and justifying. This is the reason for "ORs" in the scoring.

	The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
	How accurate, coherent, and well justified is the mathematical content?	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent are all students supported in meaningful participation in group discussions?*	*To what extent do teacher support and/or group dynamics provide access to "voice" for students?*	*To what extent does the teacher monitor and help students refine their thinking within small groups?*
1	The mathematics discussed is not at grade level; **OR** discussions are aimed at "answer getting." Explanations, if they appear, are largely procedural.	Activities or teacher intervention constrain students to activities such as applying straightforward or memorized procedures.	Some students are disengaged or marginalized, and differential access to the mathematics or to the group is not addressed.	Teacher interventions, if any, either constrain students to producing short responses to the teacher **OR** do not address clear imbalances in group discussions.	Teacher actions are simply corrective (e.g., leading students down a predetermined path) and the teacher does not meaningfully solicit or pursue student thinking.
2	Discussions are at grade level but are primarily skills-oriented, with few opportunities for making connections (e.g., between procedures and concepts) or for mathematical coherence (see glossary).	Activities offer possibilities of productive engagement or struggle with central mathematical ideas, BUT students are either left unsupported when lost, OR the teacher's actions scaffold away challenges.	All team members appear to be doing mathematics, but some are not participating in group activities; the teacher does not support their engagement in student-to-student discussion.	At least one student has a chance to talk about the mathematical content, but the teacher is the primary driver of conversations and arbiter of correctness. Students are not supported in building on each other's ideas.	Teacher solicits student thinking, but subsequent discussion does not build on nascent ideas. Teacher actions are corrective in nature, possibly by leading students in the "right" directions.
3	Explanation of and justification for central grade level mathematical ideas is coherent.	Students are supported in engaging productively with central mathematical ideas. This may involve struggle; it certainly involves having time to think things through.	Everyone in the team contributes to group or subgroup mathematical discussions, **OR** teacher moves to have all team members make meaningful contributions.	At least one student puts forth and defends his/her ideas/reasoning **AND, EITHER** students build on each other's ideas **OR** the teacher ascribes ownership for students' ideas in subsequent discussion.	The teacher solicits student thinking, **AND** subsequent discussion responds to those ideas, by building on productive beginnings or addressing possible misunderstandings.

Student Presentations

*Some episodes are in essence a conversation between teacher and student presenter(s); some conversations that involves the whole class. Scoring in the rubrics corresponds to the presence of these two different participation structures: **C** for a teacher-presenter **c**onversation, and **W** for **w**hole-class involvement.*

	The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
	How accurate, coherent, and well justified is the mathematical content?	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent does the teacher support presenters or class in engaging with the mathematics?*	*To what extent are students the source of presented ideas and response to presented ideas?*	*To what extent is students' mathematical thinking surfaced and serve as grounds for conversation?*
1	Presentation is aimed at "answer getting" without addressing underlying reasoning.	Presentation and classroom discussion focus on straightforward or familiar facts and procedures.	**(C):** Presenter(s) need support/encouragement but do not receive it; **OR** **(W):** A significant number of students appear disengaged.	Presenter role is structured by teacher/text and student is narrowly constrained in response to teacher questions.	Student reasoning is not surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
2	The mathematics presented is largely procedural; presenter(s) are not expected to explain their ideas or supported in doing so.	Presentation offers possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" these possibilities, resulting in a focus on straightforward or familiar facts and procedures.	**(C):** Teacher encourages presenters but does not provide effective scaffolding; **OR (W):** The presentation evolves into whole class activity. There is uneven participation and the teacher does not provide structured support for many students to participate in meaningful ways.	Presenters have the opportunity to demonstrate individual proficiency, without being tightly constrained by text or teacher. **BUT,** the discussions do not build on students' ideas. (*To qualify as an *idea*, what is referred to must extend beyond the tasks, diagrams, etc., that students were given.)	In presentation and discussion the teacher refers to student thinking, perhaps even to common mistakes, but specific student ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
3	The mathematics presented is relatively clear and correct, **AND** either includes justifications or explanations **OR** the teacher encourages students to focus on central mathematical ideas and explaining and justifying them.	The teacher's hints or scaffolds support presenters and/or class in "productive struggle" in building understandings and engaging in mathematical practices.	**(C):** Teacher supports presenters (if needed) in engaging, **OR (W):** The presentation evolves into whole class activity in which the teacher actively supports broad participation and/or what appear to be established participation structures result in such participation.	Student presentations result in further discussion of relevant mathematics, **OR** students make meaningful reference to other students'/groups' ideas in their presentations. (*To qualify as an idea, what is referred to must extend beyond the tasks, diagrams, etc., that students were given.)	In presentation and discussion the teacher solicits student thinking and responds to student ideas by building on productive beginnings or addressing emerging misunderstandings.

Individual Work

Student seat work is coded as N/A unless the teacher is actively circulating through the classroom and consulting with students on an ongoing basis. Note that with a stationary camera it is impossible to see individual student work. Hence, unless there is evidence from the conversation, one cannot discern student errors.

	The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
	How accurate, coherent, and well justified is the mathematical content?	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent is there equitable access to meaningful participation for all students?*	*To what extent are students the source of presented ideas; do students respond to presented ideas?*	*To what degree does the teacher monitor and help students refine their thinking as he or she circulates through the class?*
	May be N/A if there are insufficient data; or…	May be N/A if there are insufficient data; or…	May be N/A if there are insufficient data; or…	May be N/A if there are insufficient data; or…	May be N/A if there are insufficient data; or…
1	Materials are aimed at “answer getting” without addressing underlying reasoning.	Materials demand no more than applying familiar procedures or memorized facts.	A significant number of students appear disengaged and there are no overt mechanisms to support engagement.	Teacher shows or tells students how to do the mathematics, possibly correcting student work. Student ideas are not elicited or built upon.	Teacher actions are limited to corrective feedback or encouragement.
2	Materials for student work provide some affordances for coherent mathematics, but teacher support is minimal and does not exploit them.	Materials offer possibilities of conceptual richness or problem solving challenge, but teaching interventions tend to "scaffold away" the challenges.	Students appear to be working, but there are no clear mechanisms for students who want or need support or attention to receive it.	One-on-one interactions give students the opportunities to talk about their ideas and/or provide access to varied ways to engage in the mathematics.	Individual interactions provide opportunities for students to discuss their thinking, and teacher responses address such thinking explicitly (not simply correcting student work).
3	The teacher's interventions with individual students support a coherent and connected view of the mathematics.	The teacher's hints or scaffolds support students in "productive struggle" in building understandings and engaging in mathematical practices.	Teacher's and/or surrogates' attention is clearly and widely available for those students who want it, resulting in access to the mathematics.	A score of 3 is not coded *unless* the student has ample opportunity and agency to develop his/her idea interacting with the teacher, **OR** the teacher takes the student idea up for class discussion right after individual work ends.	The teacher solicits student thinking and subsequent discussions respond to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

Content Elaboration: "Robustness Criteria" for Contextual Algebraic Tasks (CATS)

	RC1: Reading and interpreting text, and understanding the contexts described in problem statements.	RC2: Identifying salient quantities in a problem and articulating relationships between them	RC3A: Generating algebraic representations of relationships between quantities	RC3B: Interpreting and making connections between representations
1	One or more terms in a task are reworded and/or defined, or a specific algebra-related term (e.g., function) in the text or lesson notes is defined	Salient quantities are identified but the relationships between quantities are not discussed (e.g., What are the slope and y-intercept?, "We know Jose's speed, we need to find the distance he travelled.")	Algebraic representation(s) is(are) generated by way of practice (e.g., writing equation for a line given two points) without attention to the relationship(s) between variables or why the representation is a good choice for the given situation.	Representations are interpreted locally or in part (e.g., relevant quantities identified but relationship between quantities is not exploited (e.g., "from the graph, when x is 4, y is what?"). There are no connections between multiple representations.
2	The context (problem scenario) is elaborated or discussed and an explicit attempt is made to ensure students understand it.	Salient quantities are identified and local relationships between quantities are discussed (e.g. at a particular point: "what is the cost of plan A for 10 hours? Of Plan B?")	Algebraic representation(s) is(are) purposefully generated with explicit attention to either the relationship between variables *or* why the representation is a good choice for the given situation.	Important global features of representations are explicated to highlight the covariation between quantities (e.g., relating the 'steepness' of a graph to a rate of change, using the representation to identify the family of functions relating the quantities) *or* connections among multiple representations are explored (e.g. focus on parameters in an equation and how they affect the features of the representations, affordances of different representations may be highlighted).
3	The teacher or students link the context (problem scenario) with algebraic concepts (e.g. rate of change, proportion, variable, expression).	General covariation of quantities is discussed (e.g." as time increases, distance stays the same"; "when x increases by 1, y increases by 2") or the relevant family of functions is identified.	Algebraic representation(s) is(are) purposefully generated with explicit attention to the relationship between variables *and* attention to why the representation is a good choice for the given situation (e.g., "let's make a graph so we can see all the possible solutions to the equation").	Important global features of representations are explicated to highlight covariation between quantities *and* connections among multiple representations are explored (e.g. focus on parameters in an equation and how the parameters affect the features of the representations); affordances of different representations may be highlighted.

Content Elaboration: “Robustness Criteria” for Contextual Algebraic Tasks (CATS)

	RC4A: Executing calculations and procedures with precision	RC4B: Checking plausibility of results	RC5A: Opportunities for Student Explanations	RC5B: Teacher instruction about Explanations	RC5C: Student Explanations and Justifications
1	Arithmetic calculations are executed accurately, and any errors are corrected.	The plausibility of a solution is passively checked (e.g. teacher poses the question, "does this answer make sense?)	An open-ended question is posed for students without explicitly soliciting an explanation or justification.	Teacher explicitly provides guidelines on what is needed *generally* for good explanations,	Student gives a short explanation that describes only procedures (whether algebraic or non-algebraic), OR the explanation is unclear.
2	Algebraic procedures (*see list) are executed accurately, and any errors are corrected.	The plausibility of a solution is actively checked without attending to context (e.g., checking that the answer makes sense with regard to a representation or calculation, but not with the context).	An explanation is explicitly requested of students, but the nature of the explanation is not specific; does not necessarily require an algebraic justification (e.g. "why?", "can you explain that?")	Teacher explicitly provides guidelines on what is *generally* needed for good explanations *and* models such behavior.	Student describes procedures, supporting them by either referring to the problem context or the underlying mathematical concepts.
3	Calculations and/or algebraic procedures are executed correctly with explicit attention to accuracy, or mistakes are caught and instruction involves guiding students to self assess and correct their calculational/procedural errors.	The plausibility of a solution is actively checked in relationship to the context (problem scenario) to make sense of the solution (i.e. to judge the meaning, utility, and reasonableness of the results; NCTM, 2000, p. 296)	An explanation is explicitly requested that focuses on algebraic reasoning (e.g. an algebraic representation, the qualitative relationship between quantities, or the problem context).	Teacher provides feedback on and/or opportunities for students to incorporate the feedback to revise *specific explanations*.	Student generates a clear algebraic explanation (e.g. draws on an algebraic representation, the qualitative relationship between quantities, or the problem context) that extends beyond explaining how to do a procedure.

*Drawing on content standard documents and related literature (CCSS-M, 2010, NCTM, 2000), we have defined algebraic procedures as including, but not being limited to:

- Substituting a value into a variable expression and evaluating
- Solving linear equations and inequalities
- Solving a proportion
- Solving a system of linear equations or inequalities through linear combinations or substitution
- Iterating recursive functions
- Finding equivalent expressions by distributing, combining like terms, etc.
- Performing arithmetic with polynomial and rational expressions
- Solving quadratic equations by factoring, completing the square, applying the quadratic formula, etc.